

Payment structure on GDP-linked bonds

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For ease of reference, terms that refer to definitions in the London Term Sheet are underlined on their first mention in this box.

We consider the fictitious country Arcadia, for which the nationally recognized Statistical Institute has published the following time series for Arcadia's seasonally-adjusted quarterly GDP in current prices, i.e. its nominal GDP.

3Q2004	100.0000	2Q2006	111.6780
4Q2004	101.4356	3Q2006	112.6280
1Q2005	103.1801	4Q2006	113.6925
2Q2005	104.5673	1Q2007	115.0475
3Q2005	106.6166	2Q2007	116.8888
4Q2005	107.8620	3Q2007	118.0834
1Q2006	110.0616	4Q2007	119.0203

We assume that the government of Arcadia has issued a 10-year GDP Bond on 5 July 2005 with a Base Interest Rate of 1%, a Principal Amount of 100 Arkadins (K\$) and a semi-annual coupon schedule. The Principal Amount of this bond is adjusted for changes in nominal GDP over the life of the bond and its semi-annual coupon payments are calculated off this adjusted principal. The Base Date for the indexation of interest and redemption amount payments has been set as 1 July 2005, i.e. two business days prior to the Issue Date.

We will now calculate the indexed principal on the second anniversary of the Base Date of this bond, i.e. on 1 July 2007. In order to achieve this, we first need to determine the Reference GDP Index for the Base Date. This is the GDP for the calendar quarter that ended six month prior to the Base Date, i.e. the fourth quarter of 2004 (4Q2004); its value is 101.4356. Then we need to determine the Reference GDP Index for the Payment Date. This is the GDP for the calendar quarter that ended six months prior to the Interest Payment Date, i.e. the fourth quarter of 2006 (4Q2006); its value is 113.6925.

Note that this indexation lag is necessary since a reliable estimate of GDP is typically released several months after the end of a given calendar quarter.

The indexed principal of the GDP-linked bond on 1 July 2007 becomes K\$112.0834 (i.e. $K\$100 \times 113.6925/101.4356$). The semi-annual coupon paid at the end of that second year is calculated as $1\%/2 \times K\$112.0834 = K\0.5604 .

At the end of the third year of the bond, the indexed principal reaches K\$117.3358 (i.e. $K\$100 \times 119.0203/101.4356$) and the semi-annual coupon amounts to K\$0.5867.

In this example, we have considered a GDP Bond with a Base Date that falls on the first calendar day of the Reference Quarter. In practice, this Base Date may also fall on other calendar days. If the base date of a new issue falls before the initial settlement date, then the bond already has an accrued coupon at issuance and a Principal Amount that needs to be indexed to an interpolated level of GDP. The same applies to the pricing of GDP Bonds in secondary market trading, which will occur on a daily basis. In order to calculate the correct Issue Price and the invoice amount

for secondary market trades, we therefore need to compute daily reference values for the Nominal GDP Index Ratio so that the Principal Amount can be adjusted for every trading day.

Following the Canadian model for inflation-linked bonds, we will calculate a Reference GDP Index (*Ref GDP*) for each settlement date, as a linear interpolation between the published quarterly nominal GDP values for two consecutive quarters in the past. We use the following formula:

$$Ref\ GDP_{Settlement\ Date} = GDP_{q-2} + \frac{(d_q - 1)}{D_q} \times (GDP_{q-1} - GDP_{q-2})$$

where:

GDP_{q-1} is the GDP for the calendar quarter ended three months prior to the Settlement Date;

GDP_{q-2} is the GDP for the calendar quarter ended six months prior to the Settlement Date;

d_q is the number of days elapsed in the Payment Quarter, i.e. the actual number of days between the end of the previous calendar quarter and the Settlement Date;

D_q is the total number of days in the Payment Quarter, using actual/actual as day-count convention.

This formula can be found in the section on Reference GDP of the London Term Sheet. It can be used to calculate the Reference GDP for the Base Date, for any Interest Payment Date or Redemption Date or for the Settlement Date of secondary market trades.

In order to determine the indexed principal of the bonds, we need two Reference GDP Index values: $Ref\ GDP_{Base\ Date}$ for the bond's Base Date, i.e. the accrual date for its first coupon payment, and $Ref\ GDP_{Settlement\ Date}$ for the settlement date. From these two index values, we can calculate a Nominal GDP Index Ratio (*IR*) to measure the principal adjustment. We use the following formula:

$$IR_{Settlement\ Date} = Ref\ GDP_{Settlement\ Date} / Ref\ GDP_{Base\ Date}$$

The full settlement price (*FP*) for a given settlement date is calculated as follows:

$$FP_{Settlement\ Date} = IR_{Settlement\ Date} \times (CP_{Trade\ Date} + AI_{Settlement\ Date})$$

where:

$CP_{Trade\ Date}$ is the clean price quoted by the broker-dealer, unadjusted for past growth;

$AI_{Settlement\ Date}$ is the accrued interest on the settlement date, unadjusted for past growth.

The accrued interest (*AI*) is calculated with the following formula:

$$AI_{Settlement\ Date} = \frac{d_s}{D_s} \times \frac{Base\ Interest\ Rate}{2} \times Principal\ Amount$$

where:

d_s is the number of days elapsed in the semi-annual coupon period, i.e. the actual number of days between previous coupon date and Settlement Date;

D_s is the total number of days in the coupon period, using actual/actual as day-count convention.

Note that this full settlement price including accrued interest will also be paid to holders of GDP-linked bonds in the case of an early redemption following the exercise of a Put Option.

We illustrate the calculation of the invoice amount for a trade in the secondary market using a GDP bond with the following characteristics:

Base Interest Rate: 1%

Base Date: 13 January 2005

First Coupon Date: 13 July 2005

We consider the purchase of K\$1 million Principal Amount on 28 August 2007 at a Clean Price of 115.25 percent of Principal Amount. First, we calculate the Accrued Interest:

Settlement Date: 30 August 2007

Previous Coupon Date: 13 July 2007

Next Coupon Date: 14 January 2008 (using the “modified following” business day convention)

Number of actual days between previous coupon date and Settlement Date (d_s): 48

Number of actual days between previous and next coupon date (D_s): 185

$$AI_{30 \text{ August } 2007} = \frac{48}{185} \times \frac{1.00\%}{2} \times K\$ 1,000,000 = \text{or } 0.12973 \text{ percent of Principal Amount} \quad K\$ 1,297.30$$

Then we calculate the Reference GDP Indices for the Base Date and for the Payment Date as well as the Nominal GDP Index Ratio:

Number of days elapsed in the Base Quarter (d_{bq}): 13

Total number of days in the Base Quarter (D_{bq}): 90

$$Ref \text{ GDP}_{13 \text{ January } 2005} = 100.0000 + \frac{(13 - 1)}{90} \times (101.4356 - 100.0000) = 100.19141$$

Number of days elapsed in the Payment Quarter (d_{pq}): 61

Total number of days in the Payment Quarter (D_{pq}): 92

$$Ref \text{ GDP}_{30 \text{ August } 2007} = 115.0475 + \frac{(61 - 1)}{92} \times (116.8888 - 115.0475) = 116.24835$$

$$IR_{30 \text{ August } 2007} = 116.24835/100.19141 = 1.16026 \text{ (rounded to the fifth decimal)}$$

With this, we calculate the total invoice amount:

$$FP_{30 \text{ August } 2007} = 1.16026 \times (115.25 + 0.12973) = 133.87049 \text{ percent of Principal Amount}$$

$$\text{Invoice Amount} = K\$ 1,000,000 \times 133.87049 \% = K\$ 1,338,704.90$$

This box has been prepared by Christian Kopf